ROLE OF BREAKUP OF LANGMUIR WAVES INTO ION-SOUND WAVES FOR AN ELECTRON BEAM INTERACTING WITH A NONISOTHERMAL PLASMA

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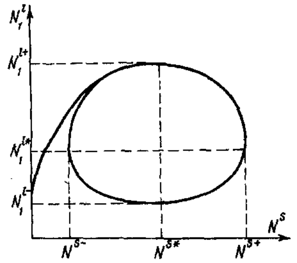
Quasi-linear theory has been applied [1–3] to nonlinear effects in the interaction of a beam of charged particles with a plasma, as has allowance for nonlinear scattering of Langmuir waves [4, 5].

When a low-density beam passes through a plasma, the effects of the nonlinear interaction usually appear at times substantially greater than the time of quasi-linear reaction, and their results amount to transformation of the spectrum arising in the quasi-linear stage.



Here I show that there are also cases in which the effects from interaction can be reduced because of breakup of Langmuir waves to Langmuir and sound waves ($l \to l' + s$) [6, 7]. The physical reason for loss of interaction effectiveness is that substantial nonlinear effects cause transfer of the oscillation energy to a nonresonant part of the spectrum, where the oscillations do not interact with the beam. If the plasma is not isothermal ($T_e \gg T_i$), the breakup $l \to l' + s$ predominates over other nonlinear processes in the range of parameters envisaged.

If the electron beam is injected into a semi-infinite plasma at $x = 0$, quasi-linear theory indicates that the beam-plasma system gives rise to a distribution with a plateau in the electron distribution function for $x \to \infty$. Let us consider a range of plasma parameters for which the quasi-linear steady level of the Langmuir waves is not attained, because of $l \to l' + s$ breakup, and no plateau is produced. Here, as previously [1–5], the treatment is one-dimensional.

We assume that the oscillations excited by the beam are propagated in the beam direction perpendicular to the boundary of the plasma ($x = 0$). The following model is used for the distribution function of the beam incident on the boundary of the plasma: $\partial f/\partial v > 0$ for a fairly narrow velocity range $v_0 - u < v < v_0 + u$ ($u \ll v_0$) with $v > 0$, while f is discontinuous at the boundaries of this range, with $\partial f/\partial v|_{v=v_0 \pm u} \to -\infty$ and $v_e \ll v_0$.

If the sound waves formed from the Langmuir waves recede to infinity (or are absorbed at the wall at $x = L$), there is a single quasi-stationary state. On the other hand, the sound waves in a plasma of length L may be reflected from the walls if the absorption is low, and then there are two possible quasi-stationary states. In the latter case, the sound waves after double reflection are propagated in the positive direction at $x = 0$ beginning at some instant t'. Sound waves in this flux lie in the resonance range with respect to breakup, so the generation of Langmuir waves by the beam will be disrupted, as will the first quasi-stationary state, since the induced breakup will at once (at $x = 0$) predominate over the generation. The resulting (second) quasi-stationary state has a high level of ion-sound waves N^s and a nearly zero level in the number of Langmuir waves $N_1{}^l \approx N_1{}^{l_0}$. This second state will be further disrupted, since scattering at the ions will cause the ion-sound spectrum $N^s(k_s)$ to be displaced from the resonant range Δk_s. The time spent in the second quasi-stationary state will then be determined by the time taken to displace $N^s(k_s)$ by Δk_s.

§ 1. Langmuir waves can [6] break up into Langmuir and ion-sound waves only if $k_1 = \omega_0/v_0 > k_0$, i.e., when

$$v_e/v_0 > {}^1/_3 \sqrt{m_e/m_i}\,, \qquad (1.1)$$

in which v_0 is the mean speed of the electrons in the beam, $k_0 = (m_e/m_i)^{1/2}/3\lambda_e$,

$$\omega_0{}^2 = (4\pi e^2 n_0/m_e)^{1/2}, \quad v_e = (T_e/m_e)^{1/2}, \quad \lambda_e = v_e/\omega_0\,.$$

The possible number of Langmuir satellites is n if $k_1 = \omega_0/v_0$ lies in the range

$$\frac{2n-1}{3\lambda_e}\left(\frac{m_e}{m_i}\right)^{1/2} < k_1 < \frac{2n+1}{3\lambda_e}\left(\frac{m_e}{m_i}\right)^{1/2}, \quad n = 1, 2, 3\ldots,$$

i.e.,

$$\frac{2n-1}{3}\left(\frac{m_e}{m_i}\right)^{1/2} < \frac{v_e}{v_0} < \frac{2n+1}{3}\left(\frac{m_e}{m_i}\right)^{1/2}. \qquad (1.2)$$

In particular, there can be only one satellite in a hydrogen plasma for $1/120 < v_e/v_0 < 3/120$, two for $3/120 < v_e/v_0 < 5/120$, etc., and six for $11/120 < v_e/v_0 < 13/120$. The model for the distribution function of the plasma + beam system becomes unsuitable for a hydrogen plasma for the larger number of satellites corresponding to $v_e/v_0 > 1/10$. The theory of one-dimensional decomposition [6] indicates that it is reasonable to consider a spectrum of nonoverlapping satellites only for $\Delta k < 4k_0$, i.e., for

$$\frac{u}{v_e} \leqslant 2\frac{k_0}{k_1}\frac{v_0}{v_e} = \frac{2v_0{}^2}{3v_e{}^2}\left(\frac{m_e}{m_i}\right)^{1/2}. \qquad (1.3)$$

The complete system of equations for this problem takes the form [3, 6]

$$\frac{\partial f}{\partial t} + v_0\frac{\partial f}{\partial x} = \gamma_1\frac{\partial}{\partial v}N_1{}^l\frac{\partial f}{\partial v},$$

$$\frac{\partial N_1{}^l}{\partial t} + V_1{}^l\frac{\partial N_1{}^l}{\partial x} = \gamma_2 N_1{}^l\frac{\partial f}{\partial v} +$$

$$+\alpha(N_2{}^lN_1{}^s - N_1{}^lN_1{}^s - N_1{}^lN_2{}^l),$$

$$\frac{\partial N_1{}^s}{\partial t} + v_s\frac{\partial N_1{}^s}{\partial x} =$$

$$= -\frac{\alpha}{2}(N_2{}^lN_1{}^s - N_1{}^lN_1{}^s - N_1{}^lN_2{}^l) - \beta_1 N_1{}^s,$$

$$\frac{\partial N_2{}^l}{\partial t} + V_2{}^l\frac{\partial N_2{}^l}{\partial x} =$$

$$= -\alpha(N_2{}^lN_1{}^s - N_1{}^lN_1{}^s - N_1{}^lN_2{}^l) +$$

$$+\alpha(N_3{}^lN_2{}^s - N_2{}^lN_2{}^s - N_2{}^lN_3{}^l)$$

. . . .

$$\frac{\partial N_n^l}{\partial t} + V_n^l \frac{\partial N_n^l}{\partial x} =$$
$$= -\alpha (N_n^l N_{n-1}^s - N_{n-1}^l N_{n-1}^s - N_{n-1}^l N_n^l) +$$
$$+ \alpha (N_{n+1}^l N_n^s - N_n^l N_n^s - N_n^l N_{n+1}^l),$$
$$\frac{\partial N_n^s}{\partial t} + v_s \frac{\partial N_n^s}{\partial x} =$$
$$= -\frac{\alpha}{2}(N_{n+1}^l N_n^s - N_n^l N_n^s - N_n^l N_{n+1}^l) - \beta_n N_n^s,$$
$$\gamma_1 = \frac{\omega_0^3}{2n_0 m_e v_0}, \quad \gamma_2 = \pi\omega_0 v_0^2, \quad \alpha = \frac{\omega_0^2 k_0}{8m_e n_0 v_e^2},$$
$$V_1^l = 3v_e\lambda_e k_1, \quad v_s = (T_e/m_i)^{1/2} = 3v_e\lambda_e k_0,$$
$$k_0 = (m_e/m_i)^{1/2}/3\lambda_e,$$
$$\beta_n = \beta_n^0 + \beta_{st}, \quad \beta_n^0 = \sqrt{1/2\pi}\, k_{sn} v_e m_e/m_i,$$
$$k_1 = \omega_0/v_0,$$
$$k_n = (-1)^n [2k_0(n-1) - k_1] \quad \text{for} \quad n = 2, 3, 4, \ldots,$$
$$k_{sn} = k_n - k_{n-1} \quad \text{for} \quad n = 1, 2, 3 \ldots \quad \lambda_e = v_e/\omega_0,$$
$$V_n^l = 3v_e\lambda_e k_n, \qquad \omega_0^2 = 4\pi e^2 n_0/m_e. \tag{1.4}$$

First we consider what parameters of plasma and beam make breakup processes important.

In the case of the pure time problem $(\partial(\ldots)/\partial x = 0)$, neglect of breakup means that the stationary level of the longitudinal waves $N_I^{l\infty} = N_1^l|_{t=\infty} = \pi m_e n_1 v_0^3/\omega_0^2$ corresponding to the maximum energy $n_1 m_e v_0 u$ of the Langmuir waves is attained after a time

$$\tau_0 = \left(\gamma_2 \frac{\partial f}{\partial v}\Big|_{t=0}\right)^{-1} = \frac{1}{\pi\omega_0} \frac{n_0}{n_1} \frac{u^2}{v_0^2}, \tag{1.5}$$

while for the breakup increment corresponding to the stationary level of the longitudinal waves we have

$$\gamma_{ls}^l \sim \alpha N_I^{l\infty} = \frac{\pi}{8} \frac{n_1}{n_0} \frac{k_0 v_0^3}{v_e^2}. \tag{1.6}$$

It is thus clear that breakup must be considered if $\alpha N_I^{l\infty} \gtrsim 1/\tau_0$, i.e., for

$$\frac{u}{v_e} \gtrsim 2\left(\frac{2k_1}{k_0}\right)^{1/2}. \tag{1.7}$$

From (1.7), with the condition for the applicability of concepts on the number of waves,

$$\max\{\alpha N_I^{l\infty},\ 1/\tau_0\} \ll \Delta\Omega = \frac{3v_e^2}{\omega_0} k_1 \Delta k, \tag{1.8}$$

we have a restriction on the beam density

$$\frac{n_1}{n_0} < \frac{48}{\pi}\left(\frac{v_e}{v_0}\right)^4, \tag{1.9}$$

which gives $n_1/n_0 < 10^{-3}$ for a hydrogen plasma.

In the case of the spatial quasi-stationary problem $(\partial(\ldots)/\partial t = 0)$, when a beam acts on a plasma half-space from the left, there is continuous renewal of the distribution function of the beam at each point. As a result, if breakup is neglected, the rise in the number of waves at any point is restricted only by the recession in the positive direction, and the stationary level in this case is

$$N_{II}^{l\infty} = N_1^l|_{x=\infty} = \frac{mn_1 v_0^3}{\omega_0^2} \frac{2\pi}{3}\left(\frac{v_0}{v_e}\right)^2. \tag{1.10}$$

The level N^{s*} (for N_2^{l*}) for equality of the increments of linear generation and breakup is

$$N^{s*} = \frac{mn_1 v_0^3}{\omega_0^2} \frac{4\pi k_1}{k_0} \frac{v_e^2}{u^2} \qquad \left(\alpha N^{s*} = \gamma \frac{\partial f}{\partial v}\Big|_{x=0}\right), \tag{1.11}$$

so it is necessary to consider breakup for $N^{s*} \lesssim N_{II}^{l\infty}$, i.e., for

$$\frac{u^2}{v_e^2} \gtrsim \frac{6\omega_0}{k_0 v_0} \frac{v_e^2}{v_0^2}.$$

Inequalities (1.3) and (1.12) are obeyed simultaneously only if

$$v_0 > v_e \sqrt[7]{81/2}\, (m_i/m_e)^{3/14}. \tag{1.12}$$

We therefore have the following system of inequalities derived from (1.1), (1.3), and (1.12) for the range of parameters within which it is necessary to consider breakup in the spatial quasi-stationary problem and where this can be done on the basis of the theory of nonoverlapping satellite bands [6]:

$$v_e \sqrt[7]{81/2}\, (m_i/m_e)^{3/14} < v_0 < 3v_e (m_i/m_e)^{1/2} \tag{1.13}$$

(for a hydrogen plasma $9v_e < v_0 < 120\, v_e$),

$$18\left(\frac{m_i}{m_e}\right)^{1/2} \frac{v_e^3}{v_0^3} \lesssim \frac{u^2}{v_e^2} \lesssim \frac{4}{9} \frac{v_0^4}{v_e^4} \frac{m_e}{m_i}.$$

The condition on n_1/n_0 is formally much more rigid for the spatial problem, because $N_{II}^{l\infty}$ exceeds $N_I^{l\infty}$ by a factor of $(v_0/v_e)^2$, and so the maximum increment of the decay $l \to l' + s$ is also larger by a factor of $(v_0/v_e)^2$. In fact, the level $N_{II}^{l\infty}$, corresponding to quasi-linear relaxation is not reached in the spatial problem, and so (1.13) is replaced simply by $1/\tau_0 \ll \Delta\Omega$ (since $\alpha N_{II}^{l\infty} \lesssim \tau_0^{-1}$):

$$n_1/n_0 \lesssim 2\,(u/v_e)^3\,(v_e/v_0)^5 < (v_0/v_e)\,(m_e/m_i)^{1/2}. \tag{1.14}$$

The breakup processes predominate over the other parallel nonlinear processes for the above range of plasma parameters. The generation increment for ion-sound waves of frequency ω_{0i} for $l \to s$ scattering is [5]

$$\gamma_{ls}^s = \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_{0i}}{nm_e v_e^2} N_1^l \frac{\omega_0^2}{v_0^2} \frac{2uv_e}{\pi v_0}.$$

Comparison of γ_{ls}^s with $\gamma_{lls}^s \sim \alpha N_1^l$ (the increment for generation of ion-sound waves in breakup) shows that breakup processes predomiate for

$$\frac{u}{v_e}\left(\frac{v_e}{v_0}\right)^3 \lesssim \frac{1}{24}\left(\frac{\pi}{2}\right)^{1/2}.$$

However, γ_{lls}^{l} (the breakup increment for longitudinal waves, is proportional to N^S, while the increment for $l \to s$ scattering is $\gamma_{ls}^{l} \sim \sim \int N^s dk$, so for

$$\gamma_{ls}{}^{l} \approx \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{2\pi} \frac{\omega_0}{nm_i v_e^2} N^{si} \frac{\omega_0 \omega_{0i}}{v_e}, \qquad \gamma_{lls}^{l} \sim \alpha N^s$$

it is necessary to have the following condition for breakup to predominate:

$$\frac{\gamma_{ls}{}^{l}}{\gamma_{lls}^{l}} \simeq \frac{N^{si}}{N^s} 6 \left(\frac{2}{\pi}\right)^{1/2} \ll 1 .$$

This condition is readily shown from (1.3) and (1.7) to be always satisfied with a margin of at least an order of magnitude.

Comparison of the increment for $l \to l$ shift at ions for $T_e \gg T_i$ with γ_{lls}^{l} (breakup increment) gives us a condition for predominance of breakup:

$$\frac{v_0^2}{v_e^2} \left(\frac{T_i}{T_e}\right)^2 \frac{3}{2} \left(\frac{m_e}{m_i}\right)^{1/2} \ll \frac{u}{v_e}, \qquad (1.15)$$

which is always obeyed when (1.7) is obeyed.

Note that $l \to l$ scattering at electrons is less strong than that at ions.

Growth of breakup does not necessarily have to occur in a unique fashion, because the onset is substantially dependent on suitable centers. For instance, let a sound center arise at x_0 at time t_0 when the stationary quasi-linear picture has been attained at a sufficiently high level N_1^l [3]. Then breakup starts near x_0 and expands in both directions from that point with the group velocities v_s (ion sound) and V_2^l (Langmuir satellites). Then N_1^l in the range $(0, x_0)$ is reduced in a time $\sim x_0/|V_2^l|$, and correspondingly the beam for this time in the region $x > x_0 + x_0|V_1^l/V_2^l|$ proceeds with an almost unchanged distribution function. Thus N_1^l in that region can exceed even the quasi-linear stationary level for some time. However, subsequent growth of the breakup gradually leads to establishment of the quasi-stationary state.

§ 2. Consider the solution to (1.4) in the quasi-stationary case $(\partial(\ldots)/\partial t = 0)$.

As N_2^l has a maximum on account of nonlinear interaction of the beam with the plasma, there is a time t^* such that $N_2^l \ll N^S$ (N^S only increases, because the sound spectrum in this approximation is not subject to breakup, and the phase velocity of the sound waves is less than the phase velocity of the Langmuir waves involved).

Then, if the plasma and beam parameters allow there to be two or more satellites, we can always neglect N_2^l in the equations for f, N_1^l, and N^S. The chain of equations for all possible satellites then automatically terminates, and we get for the quasi-stationary state that

$$v_0 \frac{\partial f}{\partial x} = \gamma_1 \frac{\partial}{\partial v} N_1{}^l \frac{\partial f}{\partial v},$$

$$V_1{}^l \frac{\partial N_1{}^l}{\partial x} = \gamma_2 N_1{}^l \frac{\partial f}{\partial v} - \alpha N_1{}^l N^s,$$

$$v_s \frac{\partial N^s}{\partial x} = \frac{\alpha}{2} N_1{}^l N^s . \qquad (2.1)$$

Eliminating N^S from (2.1), on the assumption that $N^S(0, t) \equiv N_0{}^S$, we get for the general case that

$$\frac{\partial}{\partial x}\left[\frac{V_1{}^l}{\gamma_2} \frac{\partial \ln N_1{}^l}{\partial x} + \frac{\alpha}{\gamma_2} N_0{}^s \exp\left(\frac{\alpha}{2v_s}\int_0^x N_1{}^l dx\right)\right] = \frac{\gamma_1}{v_0} \frac{\partial^2}{\partial v^2}\left\{N_1{}^l\left[\frac{V_1{}^l}{\gamma_2} \frac{\partial \ln N_1{}^l}{\partial x} + \frac{\alpha}{\gamma_2} N_0{}^s \exp\left(\frac{\alpha}{2v_s}\int_0^x N_1{}^l dx\right)\right]\right\} \qquad (2.2)$$

In this paper we consider only the case in which $l \to l' + s$ breakup destroys the quasi-linear relaxation (with its plateau on the distribution function), so the influence of the waves on the distribution function is small and the generation increment may be considered as constant:

$$\frac{\partial f}{\partial v} \simeq \frac{\partial f}{\partial v}\bigg|_{x=0} = \frac{1}{2} \frac{n_1}{n_0} \frac{1}{u^2} = A_0 .$$

Solution of the last two equations in (2.1) amounts to the quadrature

$$\gamma x = \int_{N_0^s}^{N^s} \frac{dN^s}{N^s}\left[N_1{}^{l0} + \frac{a}{\gamma} \ln \frac{N^s}{N_0{}^s} - \frac{b}{\gamma}\left(\frac{N^s}{N_0{}^s} - 1\right)\right]^{-1}$$

$$N_1{}^l = N_1{}^{l0} + \frac{a}{\gamma} \ln \frac{N^s}{N_0{}^s} - \frac{b}{\gamma}\left(\frac{N^s}{N_0{}^s} - 1\right), \qquad (2.3)$$

in which

$$a = \frac{\pi}{6} \frac{n_1}{n_0} \frac{v_0^3}{v_e^3} \frac{v_e^2}{u^2} \frac{1}{\lambda_e},$$

$$b = \frac{\omega_0{}^2 k_0 v_0 N_0{}^s}{24 m_e n_0 v_e{}^4}, \qquad \gamma = \frac{\omega_0{}^3}{48 m_e n_0 v_e{}^4}.$$

However, it is more convenient to reduce these equations to

$$\frac{d \ln N_1{}^l}{dx} = a - b \exp\left(\gamma \int_0^x N_1{}^l dx\right). \qquad (2.4)$$

This shows that $N_1{}^l = N_1{}^{l_0} \exp(ax)$, initially for $x < c$, in which c is defined by

$$a = b \exp\left(\gamma \int_0^c N_1{}^l dx\right).$$

The value $N_1^{l\max}$ at $x = c$ is followed for $x > c$ by a fairly rapid fall. We can estimate c from

$$a = b \exp\left[\gamma \int_0^c N_1{}^{l0} \exp(ax)\, dx\right], \quad N_1{}^{l0} \equiv N_0{}^l,$$

which with $a/N_0{}^l\gamma \gg 1$, $\ln(a/b) \gg 1$ gives

$$c = \frac{1}{a} \ln\left[\frac{a}{\gamma N_0{}^l} \ln(a/b)\right] = \frac{1}{a} \ln(N_1^{l\max}/N_0^l). \qquad (2.5)$$

Then

$$N_1^{l\max} = N_1^{l0} \exp(ac) = \frac{a}{\gamma} \ln \frac{a}{b} =$$

$$= \frac{8\pi n_1 m_e v_0^3}{\omega_0^2}\left(\frac{v_e}{u}\right)^2 \ln \frac{4\pi n_1 m_e v_0^2 v_e^2}{k_0 \omega_0 N_0^s u^2}. \qquad (2.6)$$

The condition, for this case to occur, is $N_1^{l\max} \ll N_{II}^{l\infty}$, where the linear increment may be considered as constant, i.e.,

$$\frac{u}{v_e} > \frac{v_e}{v_0}(12 \ln (a/b))^{1/2}, \qquad \frac{a}{b} = \frac{4\pi n_1 m_e v_0^2 v_e^2}{k_0 \omega_0 N_0^s u^2}. \qquad (2.7)$$

Violation of (2.7) corresponds to the case in which nonlinear processes follow the quasi-linear ones. As we are interested in nonlinear suppression of the quasi-linear relaxation, we will not consider the case in which (2.7) is not obeyed. Condition (2.7), taken with (1.12) and (1.15), sets a lower bound to u/v_e; also, (2.7) is compatible with (1.3) only when

$$\frac{v_e}{v_0} < \left[\frac{1}{3}\left(\frac{m_e}{m_i}\right)^{1/2} \frac{1}{\sqrt{3 \ln (a/b)}}\right]^{1/3}, \qquad (2.8)$$

which implies that $v_0 \gtrsim 10 v_e$ for a hydrogen plasma. Then (1.13) is replaced by the following conditions for the range of application of (2.3), subject to (2.7) and (2.8):

$$\max\left\{\sqrt{18}\left(\frac{m_i}{m_e}\right)^{1/4}\left(\frac{v_e}{v_0}\right)^{3/2},\right.$$

$$\left.\frac{v_e}{v_0}\left(12 \ln \frac{a}{b}\right)^{1/2}\right\} \ll \frac{u}{v_e} < \frac{2}{3}\frac{v_0^2}{v_e^2}\left(\frac{m_e}{m_i}\right)^{1/2}, \qquad (2.9)$$

$$\max\left\{\left(\frac{81}{2}\right)^{1/7}\left(\frac{m_i}{m_e}\right)^{3/14},\right.$$

$$\left.\left[3\left(3 \ln \frac{a}{b}\right)^{1/2}\left(\frac{m_i}{m_e}\right)^{1/2}\right]^{1/3}\right\} < \frac{v_0}{v_e} < 3\left(\frac{m_i}{m_e}\right)^{1/2}. \qquad (2.10)$$

Further, the condition is as follows for the application of the concept of the number of quanta $2\pi/k_1 = \lambda \ll c$:

$$\frac{n_1}{n_0} \ll \frac{1}{\pi}\left(\frac{v_e}{v_0}\right)^4\left(\frac{u}{v_e}\right)^2 \ln (N_1^{l\max}/N_0^l), \qquad (2.11)$$

and so (2.11) is obeyed when (1.14) is obeyed.

The time to attain the spatially stationary picture is

$$\tau_1 = \frac{c}{v_s}, \qquad c = \frac{1}{a} \ln (N_1^{l\max}/N_0^l), \qquad (2.12)$$

in which $v_s = (T_e/m_i)^{1/2}$ is the speed of sound in the plasma.

§ 3. If in some way it is possible to produce a strong sound-wave flux

$$\left(N^s > N^{s*} = \gamma_2 \frac{\partial f}{\partial v} \Big/ \alpha\right)$$

in the region of resonant wave numbers at x = 0 in the positive direction, this quasi-stationary state will be suppressed, as will any generation of longitudinal waves by the beam, and we get a quasi-stationary state with $N_1^l \equiv N_0^l$. One of the cases in which such suppression can occur is a bounded plasma of size $L > a^{-1}\ln(N_1^{l\max}/N_0^l)$ with mirror walls; then (2.12) gives the time taken to produce the first quasi-stationary state, which is suppressed by the sound waves after a time on the order of

$$\tau_2 = 2L/v_s. \qquad (3.1)$$

If the linear attenuation of the sound is negligible, the second quasi-stationary state can be suppressed, for example, by nonlinear displacement (at ions) of the sound waves toward smaller wave numbers by $\Delta k_s = 2\Delta k_1$, in which $\Delta k_1 = 2u\omega_0/v_0^2$, after which the first quasi-stationary state recurs. These two states then continue to alternate. The level $N^{s\max}$, which is attained for $x \gg c$, in the first quasi-stationary state is several times larger than

$$N^{s*} = \gamma_2 \frac{\partial f}{\partial v} \Big/ \alpha = \frac{m n_1 v_0^3}{\omega_0^2}\left(\frac{v_e}{u}\right)^2 \frac{8\pi k_1}{k_0}.$$

We can estimate [8] the rate of displacement of the ion-sound spectrum in k-space:

$$\frac{\delta k}{\delta t} \sim \frac{A_0 T_e v_s}{8\pi^3 n_0 e^2} k^6 N^s, \quad A_0 = \frac{T_i \theta_0^2 \omega_{0i}}{2 v_s T_e^2 n_0},$$

$$W_s \approx (2\pi)^{-3}\int N^s(k_s)\, \pi\omega_s \theta_0^2 k_s^2 dk_s. \qquad (3.2)$$

The following is the time for displacement of the sound-wave spectrum by Δk_s, and hence, the time for disruption of the first stationary state:

$$\tau_3 \sim \Delta k_s \Big/ \frac{\delta k}{\delta t} = \frac{\Delta k_s^2 v_s n_0 m_i T_e}{2 T_i k_s^3 W^s} = \frac{k_1 k_0 m_i n_0 u^3 \omega_0}{2 k_s^4 T_i n_1 v_0^3}. \qquad (3.3)$$

The following are the conditions that cause alternate termination of the first and second quasi-stationary states:

$$\beta_1 < 1/\tau_3 \ll v_s/2L < v_s a/2 \ln (N^{l\max}/N_0^l) \qquad (3.4)$$

in which β_1 is the linear damping for the sound.

An analogous picture may also occur if the second quasi-stationary state is suppressed by linear damping (or absorption at the boundaries x = 0, L) for the sound waves, when $N^s < N^{s*}$ for x = 0:

$$1/\tau_3 < \beta_1 \ll v_s/2L < v_s a/2 \ln (N_1^{l\max}/N_0^l). \qquad (3.5)$$

If $\beta_1 = \beta_1^0 = (\pi/2)^{1/2} k_s v_e m_e/m_i$ (Landau damping) for a given noise level, we may get smoothing of the part of the electron distribution responsible for linear absorption, and then the energy $N^s\omega_s\Delta k_s(2\pi)^{-1}$ becomes comparable with $mv_0\Delta v f^e \Delta v n_0$, and β_1^0 becomes zero. The following is the level N^s at which a plateau appears in the electron distribution and $\beta_1^0 \to 0$:

$$N_p^s = 4\sqrt{2\pi}\, m_e n_0 v_e^3 \frac{m_e k_s u}{m_i k_1 \omega_0^2}\left(\frac{v_e}{v_0}\right)^3.$$

There is no linear damping at this N^s and above. In our case the ratio

$$\frac{N^{s*}}{N_p^s} = \frac{\sqrt{2\pi}\, n_1}{2 n_0} \frac{m_i}{m_e} \frac{\omega_0}{k_s v_e}\left(\frac{v_0}{v_e}\right)^2\left(\frac{v_0}{u}\right)^3 \frac{k_1}{k_s} \gg 1. \qquad (3.6)$$

Then it is necessary to assume that $\beta_1 = \beta_1^0 + \beta_{st}$, in which β_{st} is due to collisions.

Calculation of the initial (thermal) levels [9] gives

$$N_0^l = \frac{m_e \omega_0}{27\pi}, \qquad N_0^s = \frac{m_e \omega_0^2}{36\pi v^3 k_s}.$$

As an example, consider a hydrogen plasma with $n_1/n_0 = 10^{-4}$, $n_0 = 3 \cdot 10^{10}$ cm^{-3}, $k_1 = 10\ k_0$ (five satellites) and

$$k_s = 2\ (k_1 - k_0) = 18\ k_0,\quad \omega_0 = 10^{10}\ \text{sec}^{-1},$$

$$v_e = 10^8\,\text{cm/sec},\quad u/v_e = 2,\quad T_e\,/\,T_i = 10^2,$$

$$\ln(a/b) = 11,\quad a = 2.9\,\text{cm}^{-1},\quad \ln(N_1^{l\,\max}/N_0^l) = 17.5,$$

$$c = 6.0\text{cm},\quad \lambda_1 = 2\pi/k_1 = 0.8\text{cm}.$$

In that case, the time for formation of the first quasi-stationary state is $\tau_1 = 2.6$ μsec, the time for suppression of this state (with L = = 10 cm) is $\tau_2 = 8.6$ μsec, and the time of existence of the second quasi-stationary state is not more than $\tau_3 = 500$ μsec.

§ 4. When linear damping of the sound waves due to collisions is incorporated, $\beta_1 = \beta_{st}$ and $N_2^l \ll N^s$, i.e., when n > 1, the initial system has, for $\partial f/\partial v$ = constant, solutions with closed phase loci similar to those shown in the figure. The process corresponding to the lower part of the curve is very slow. We have the following for the spatial problem:

$$V_1^l \frac{\partial N_1^l}{\partial x} = \gamma_2 N_1^l \frac{\partial f}{\partial v} - \alpha N_1^l N^s,$$

$$v_s \frac{\partial N^s}{\partial x} = \frac{\alpha}{2} N_1^l N^s - \beta_1 N^s. \qquad (4.1)$$

Then

$$\frac{V_1^l}{2v_s} \frac{\partial N_1^l}{\partial N^s} \frac{N^s}{N_1^l} = \frac{N^{s*} - N^s}{N_1^l - N_1^{l*}},$$

$$N_1^{l*} = \frac{2\beta_1}{\alpha},\quad N^{s*} = \frac{\gamma_2}{\alpha} \frac{\partial f}{\partial v}\Big|_{x=0}. \qquad (4.2)$$

The integral of

$$N_1^l - N_1^{l\,\max} - N_1^{l*} \ln(N_1^l\,/\,N_1^{l\,\max}) =$$

$$= \frac{2k_0}{k_1} [N^{s*} \ln(N^s\,/\,N^{s*}) - (N^s - N^{s*})] \qquad (4.3)$$

for $N^s = N^{s*}$ defines the two values

$$N_1^{l\,(1)} = N_1^{l+} = N_1^{l\,\max},$$

$$N_1^{l\,(2)} = N_1^{l+} e^{-1/c} = N_1^{l-},\qquad c = N_1^{l*}\,/\,N_1^{l+}.$$

For $N_1^l = N_1^{l*}$, the integral of (4.3) defines similarly the two values

$$y_1 = N^{s-}\,/\,N^{s*},\qquad y_2 = N^{s+}\,/\,N^{s*}$$

which are solutions to

$$\ln y + 1 - y = \frac{k_1}{2k_0} \frac{N_1^{l+}}{N^{s*}} (c - 1 - c \ln c). \qquad (4.4)$$

The upper part of the graph corresponds to the time $\tau_1 = c/v_s$ (§2), while the increment for the lower part is $\gamma \sim \alpha N_1^l/2 - \beta_1$ (with $\beta_1 > (\alpha/2) N_1^l$) and in any case is not greater than β_1, and so the attenuation length is not less than $L_3 = v_s/\beta_1$ (correspondingly, $\tau_3 = L_3/v_s = 1/\beta_1$). The above periodicity can occur for a plasma with a size $L > L_3$, while the stationary pattern of § 2 will occur in a semi-infinite plasma.

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